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Brane World Scenarios and the Cosmological Constant

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Abstract

Brane world scenarios offer a way of ensuring that a Poincare invariant four dimensional world can emerge, without fine tuning, as a solution to the equations of motion of an effective action. We discuss the different ways in which this happens, and point out that the underlying reason is that there is a contribution to the effective cosmological constant which is a constant of integration, that maybe adjusted to ensure a flat space solution. Basically this is an old idea revived in a new context and we speculate that there may be string scenarios that provide a concrete realization of it. Finally we discuss to what extent this is a solution to the cosmological constant problem.

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1 Introduction

Brane world scenarios are based on the hypothesis that the three space dimensional world that we appear to be living in is a brane that is embedded in a higher dimensional world[1]. Most of the work on this has been of a phenomenological nature and not many attempts have been made to justify the postulates within a well defined framework for (higher dimensional) quantum gravity such as string theory. Nevertheless this activity is "string inspired", in that an obvious candidate for such a world is a collection of D-branes on which (at least in principle) the standard model can live. For most of this paper we will not worry about a string realization though towards the end we will suggest some possibilities.

The main issue that we are concerned with here, is that of obtaining flat 3+1 dimensional solutions to the equations of the effective higher dimensional theory in a natural way (i.e. without fine tuning). We will show that there are situations where flat brane solutions can be obtained by choice of integration constant[2]. In this respect this mechanism is a realization whithin the brane world context of an old idea going back to [7][8],[9],[10],[11],[12][3]. To set the stage for the brane world calculations we will first review this argument.

Consider an effective theory describing our four dimensional world at low energies of the form

$$S = \frac{1}{2\kappa^2}\int\left(\sqrt{-G}R - \frac{1}{2}F_4\wedge^* F_4\right) + S_m(G,\psi), \tag{1}$$

where S_m is the matter action and F_4 is a four form field strength satisfying the Bianchi identity $dF_4 = 0$. The equations of motion from this action are,

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R - \frac{1}{2.4!}(4F_{\mu\ldots}F_{\nu}^{\ldots} - \frac{1}{2}G_{\mu\nu}F^2) &= 2\kappa^2 T_{\mu\nu} \\ d^*F &= 0. \end{aligned} \tag{2}$$

In the above $T_{\mu\nu}$ is the matter stress tensor and we have ignored the matter equations of motion which will not play any role in this paper. Now the four form equation of motion and Bianchi identity have the solution,

$$F_4 = \mu^* 1 \tag{3}$$

where μ is a constant and the second factor is the volume form. When this is substituted into the first equation one gets

$$R_{\mu\nu} = G_{\mu\nu}(2\kappa^2 V_0 + \frac{\mu^2}{4}) \tag{4}$$

[1]This is an old idea [1] that has been revived recently in a string inspired context in [2][3].

[2]After this work was substantially completed several papers appeared which obtain a one flat brane solution by choice of integration constant [4][5][6]. We will comment on these works in the conclusion.

[3]There are also unimodular gravity scenarios that appear to have been first discussed in [13]and are also discussed in [14]. However these do not fit in naturally in a string picture.

Here V_0 is the effective cosmological constant generated in the matter sector. Clearly if this is negative then the integration constant μ can always be chosen so as to get flat space. The question is what is the significance of this result. We should empahsize here that from the perspective of a four dimensional theory the integration constant μ can take any real value. However when the theory is embedded in a higher dimensional theory such as string theory which admits branes which are sources of the four form field, μ is quantized and this mechanism would not work. We will encounter such a situation later on when we discuss what happens in string theoretic brane world scenarios.

First note that if one wants to argue that a flat space solution can be obtained, even in the presence of quantum corrections to the matter action (ignoring gravity sector fluctuations), then one should replace the classical matter action S_m by the quantum effective action Γ_m. V_0 is now dependent on the RG scale and so the integration constant needs to be renormalization scale dependent in order to get flat space at every scale. Of course such a constant can be chosen at will but to solve the cosmological constant problem [14] the question of why out of the real line of values of this integration constant, one particular value (or one value at each scale) gets chosen should also be answered. At present there is no clear answer to this and we will discuss this question further in section IV.

Nevertheless one may take the point of view that replacing a fine tuning problem with a choice of integration constant is progress, since one is not adjusting a parameter in the Lagrangian. In fact in string theory there are no parameters to adjust and one might well need a mechanism like this to get flat space solutions after supersymmetry breaking. So it might still be worth investigating whether such mechanisms are available there.

In the next section we will motivate a brane world scenario from a bottom up approach as opposed to a top down string approach by asking whether the RG scale in four dimensions can be thought of as a fifth dimension. In section three we will discuss explicit embeddings of branes in five dimensions and discuss how the flat one and two brane solutions emerge. In fact in the string theory case we will argue that the natural scenario is a two-brane one. From the five dimensional point of view this requires a fine tuning of a parameter in the bulk potential, but we will argue that there are compactifications of string theory in which this parameter is (from the ten dimensional point of view) an integration constant. In the concluding section we discuss the problem of justifying the choice of integration constants that leads to flat branes.

2 Renormalization Group Flow in External Gravity.

Let us consider the quantum theory corresponding to the classical action S_m. The fields ψ could stand for the full set of standard model fields and we will also include a dilaton ϕ in order to make the connection later on to string theory. We are going to do semi-classical dilaton-gravity. In other words the dilaton gravity sector is treated classically while the standard model fields are treated quantum mechanically. The quantum theory

is defined by the functional integral,

$$e^{iW[G,\phi]} = \int [d\psi] e^{iS_m[G,\phi,\psi]}. \tag{5}$$

In order to define the quantum theory in a general gravitational background, a proper time cutoff propagator[15]

$$K_\epsilon^{-1} = \int_{\epsilon_0}^{\epsilon} e^{-Ks} ds \tag{6}$$

is introduced with K being the kinetic operator. Here ϵ_0 may be regarded as the ultra-violet cutoff (taken for instance to be the string scale) and ϵ may either regarded as a renormalization scale or the scale defining a Wilsonian effective action. Using also the technique of Riemann normal coordinate expansions, one can derive in principle the quantum effective action in a systematic way preserving general covariance. The quantum action can therefore be written in a derivative expansion as

$$W[G,\phi] = \int d^4x \sqrt{-G} (\Phi(\phi,\epsilon) R - Z(\phi,\epsilon)(\nabla\phi)^2 + V(\phi) + ... \tag{7}$$

where the elipses represent higher derivative terms. We have indicated the explicit dependence on the RG scale. There would also of course be implicit dependence since the external fields G, ϕ,like the couplings of the theory will aquire ϵ dependence. Also we have set all expectation values of standard model fields to their values solving the equations of motion (at this point the functional W is in fact equal to the 1PI effective action Γ) and have been suppressed. The RG equation reads,

$$\frac{dW}{d\epsilon} = \frac{\partial W}{\partial \epsilon} + \beta_\lambda \frac{\partial W}{\partial \lambda} + \beta_{\mu\nu} . \frac{\delta W}{\delta G_{\mu\nu}} + \beta_\phi . \frac{\delta W}{\delta \phi} = 0 \tag{8}$$

where the λ are the couplings in the theory with associated beta function β_λ and the other betas are the analogous beta functions for the metric and phi field (which are to be treated as generalized couplings). When the classical action for gravity and the F_4 field are added to the above quantum action we again get an action of the form of (1) (plus higher derivative terms) but with couplings which depend on ϕ and the RG scale ϵ. After a Weyl transformation this can be written as

$$S = \frac{1}{2\kappa^2(\epsilon)} \int \left(\sqrt{-G} R - \tilde{Z}(\phi,\epsilon)(\nabla\phi)^2 - \frac{1}{2} U(\phi,\epsilon) F_4 \wedge^* F_4 - 2\kappa^2 V(\phi,\epsilon) + ... \right) \tag{9}$$

The previous argument still goes through with slight modifications. For instance now the four form equation is replaced by $d^*(U(\phi)F_4) = 0$ which is solved by

$$F_4 = \mu U^{-1*} 1 \tag{10}$$

(which also satisfies the Bianchi identity). But the main result remains unchanged. The cosmological constant is an integration constant which can be chosen (in a RG scale dependent way) so as to get the effective cosmological constant to be zero. The

argument is robust under renormalization of the standard model since it did not depend on particular functional forms of Z, u or V. The problem of justifying the choice of integration constant however remains.

Let us now ask the question under what circumstances can the RG scale of the four dimensional theory be interpreted as a fifth dimension. In [16] the argument was made that the five dimensional Hamilton-Jacobi equation can be interpreted as a four dimensional RG equation. Here we ask the opposite question; under what conditions can the latter be interpreted as a five dimensional gravity theory?

Consider the following expression constructed in terms of the quantum effective action W defined in (7),

$$\begin{aligned}\frac{1}{\sqrt{-G}}\frac{1}{3}\left(G^{\mu\nu}\frac{\delta W}{\delta G^{\mu\nu}}\right)^2 &- \frac{\delta W}{\delta G^{\mu\nu}}\frac{\delta W}{\delta G_{\mu\nu}} - \frac{1}{2}\left(\frac{\delta W}{\delta\phi}\right)^2 \\ &= \sqrt{-G}(\tilde{V}(\phi,\epsilon) + \frac{1}{\kappa^2(\phi,\epsilon)}R + M(\phi,\epsilon)(\nabla\phi)^2 + \ldots \qquad (11)\end{aligned}$$

The right hand side is just a consequence of general covariance and the ellipses stand for higher derivative terms. The particular form of the expression on the left hand side is of course chosen to agree with the corresponding expresion in the Hamilton-Jacobi equation of five dimensions [16]. Under what conditions can W be interpreted as a classical five dimensional action? Clearly this is possible if the explicit dependence on ϵ is absent.[4] It is possible that this is the case in $\mathcal{N} = 4$ SU(N) Yang-Mills theory (at least in the large N limit) and this would then be an explanation of the AdS/CFT conjecture [18].

Now the semi-classical theory of quantum fields is obtained after one adds a classical action and one then gets the action (9). Let us set the F_4 terms to zero for the moment and ask what happens to the cosmological constant. Let $\phi = \phi_0(\epsilon)$ be a constant field satisfying $\frac{\partial V(\phi,\epsilon)}{\partial\phi} = 0$. The gravity equation then gives $R_{\mu\nu} = \frac{1}{2}V(\phi_0(\epsilon),\epsilon)G_{\mu\nu}$. Clearly if the explicit dependence of V on ϵ is absent then ϕ_0 is ϵ independent and so is the Ricci curvature so that if one has tuned the minimum of V to zero at some scale (for instance $\epsilon = \epsilon_0$) then one will get flat space at all scales. But the issue is precisely for what theories in four dimensions is the statement of independence from ϵ valid. With sufficient supersymmetry this could be the case. But with $\mathcal{N} = 1$ SUSY although the superpotential is not renormalized the Kahler potential is, so that the potential for ϕ will in general depend explicitly on ϵ, though of course in this case one does not expect renormalization of the minimum of the potential. Thus in order to have a flat space solution at any RG scale one would in general need something like the mechanism discussed earlier.

Now it may be the case that, the absence of explicit dependence on ϵ in W, while a sufficient condition for the five dimensional interpretation, is not be a necessary one. In other words there could be a cancellation of the epsilon dependence on the LHS of (11) amongst the different terms so that the RHS is ϵ independent. In this case just the mere fact that a five dimensional interpretation (as in the AdS/CFT case) exists, is

[4]It should be noted that this explicit dependence includes the dependence on ϵ through the renormalization of the flat space couplings as well. i.e. it corresponds to the first two terms of (8).

no gurantee of RG invariance of the four dimensional cosmological constant[5]. In other words the logic cannot be reversed. The absence of explicit dependence of W on ϵ (which implies in particular that the cosmological constant is not renormalized) is a sufficient condition for a five dimensional interpretation, but the latter does not imply that the former is the case.

3 Brane World Scenarios

In the previous section we discussed the assumptions that would lead us to interpret the RG scale as a fifth coordinate and thus four dimensional semi-classical gravity as a five dimensional gravity theory. Here we will explicitly treat the four dimensional theory as living on a brane in five dimensions. It is important to keep in mind the distinction between the two cases. In the first case the five dimensional theory (as for example in the AdS/CFT case) is simply a dual representation of the four dimensional quantum effective action. In the present case the underlying theory is five (or more dimensional) and the standard model is confined to a 3-brane living in it. This may perhaps be realized in string theory as for example a type IIB orientifold (compactified on some compact 5-manifold) with D3 branes and we shall discuss this further at the end of this section.

Using only general covariance, and keeping only two derivative terms, the most general five dimensional action of gravity coupled to a scalar field is,

$$S[G,\phi] = \int d^5x\sqrt{-G}(R - Z(\phi,\epsilon)(\nabla\phi)^2 + V(\phi) + ...) \tag{12}$$

If this originates from the string theory example mentioned above, the potential V may come from the F_5 terms that occur there, just like the F_4 terms in equation (9), after using the solution to the equation of motion for the F_5 field[6].

Let us take the coordinates to be $x^M,\ M = 0, 1, ..4$ with the fifth coordinate $x^4 = u$. Now we insert 3-branes transverse to the direction u at the points $u = u_i$. We choose the static gauge so that the embedding functions are $x^\mu(\xi) = \xi^\mu,\ \mu = 0, ..3$ and ignore their fluctuations. The effective action(s) coming from integrating the "standard model" quantum fields (and hidden sector fields if there is more than one brane) will then take the form.

$$-\sum_i \int_{u=u_i} T_i(\phi,\epsilon)\sqrt{-G_{4(i)}} \tag{13}$$

There will also be derivative terms but since we are interested in solutions with flat metrics and constant fields in 4d, they are irrelevant to our discussion. The field equations for the system are then obtained by extremizing the sum of the two actions (12,13).

Now as in [19],[20],[21] we look for solutions that give flat space and constant ϕ field on the brane. So we write

$$ds^2 = e^{2\omega(u)}\eta_{\mu\nu}dx^\mu dx^\nu + du^2$$

[5]Some discussion of the consequences of this are found in [17].

[6]Thus in the notation of (9) and the sentence below it, (rewritten for five dimensions) V in the above would be $\frac{1}{2}\mu^2 U(\phi)^{-1}$

$$\phi = \phi(u). \tag{14}$$

The equations of motion then take the following form, (writing $\frac{d}{du} \equiv'$)

$$\begin{aligned} 6\omega'^2 &= \frac{1}{2}Z(\phi)\phi'^2 - V(\phi) \\ 3\omega'' + 6\omega'^2 &= -\frac{1}{2}[Z(\phi)\phi'^2 + V(\phi)] - \frac{1}{2}\sum_i 2\kappa^2 T_i(\phi,\epsilon)\delta(u-u_i) \\ Z(\phi'' + 4\omega'\phi') + \frac{1}{2}\phi'^2\frac{dZ}{d\phi} &= \frac{1}{2}V'(\phi) + \kappa^2\sum_i \frac{dT_i}{d\phi}\delta(u-u_i) \end{aligned} \tag{15}$$

The delta functions (due to the presence of the branes) imply that ω' and ϕ' are discontinuous at the branes and satisfy the matching conditions

$$\begin{aligned} 3(\omega'(u_i+0) - \omega'(u_i-0)) &= -\kappa^2 T_i|_{u_i} \\ Z|_{u_i}(\phi'(u_i+0) - \phi'(u_i-0)) &= \kappa^2\frac{dT_i}{d\phi}|_{u_i} \end{aligned} \tag{16}$$

It should be noted that general covariance would imply that the scalar field equation should be satisfied when Einstein's equations (the first two in the above set (15)) are satisfied. In the presence of the branes (which break the five dimensional general covariance) the consistency of the third with the first two implies a condition

$$(\phi'\frac{dT_i}{d\phi})|_{u_i} = 4(\omega' T_i)|_{u_i} \tag{17}$$

where we may define $\phi'(u_i) = \frac{1}{2}(\phi'(u_i+0) + \phi'(u_i-0))$ and similarly with $\omega'(u_i)$. In fact this condition is the same as what one would get from requiring that the potential be continuous at $u = u_i$ and using the first equation of (15). However when $\phi'(0), \omega'(0)$ are zero (as is the case if we impose a Z_2 symmetry under $u \to -u$) (17) is trivially satisfied.

Let us first consider solutions with one brane located say at $u = 0$. Also suppose that the bulk potential is of the form

$$V = \left(\frac{\partial W}{\partial\phi}\right)^2 - \frac{4}{3}W^2. \tag{18}$$

where $W = W(\phi)$ may be considered to be a sort of superpotential. This form for V arises naturally in gauged supergravities and appears to be a necessary condition for the existence of a solution [21],[16],[22]. In this case the solutions for the warp factor and the scalar field can be obtained from [21],[22],

$$3\omega' = -W(\phi), \qquad \phi' = \frac{dW}{d\phi} \tag{19}$$

which can be solved by quadratures. Given these bulk solutions then the existence of a flat brane is guaranteed provided the matching condition is satisfied. But this is just a matter of choosing integration constants.

Let us discuss this further. We will impose a Z_2 symmetry as in [19],[21]. This might be a useful constraint in that the most likely string realization of the brane world scenario is probably a type II orientifold. Thus we impose

$$\omega(u) = \omega(-u), \quad \phi(u) = \phi(-u). \tag{20}$$

The matching conditions (16) become (for the brane at $u = 0$),

$$\begin{aligned} 3\omega'|_{u=0+} &= -\frac{1}{2}\kappa^2 T_0(\phi)|_{u=0+} \\ Z(\phi)\phi'|_{u=0+} &= +\frac{1}{2}\kappa^2 \frac{dT_0}{d\phi}(|_{u=0+} \end{aligned} \tag{21}$$

The two second order equations for ω and ϕ would have two integration constants each. However the first equation of (15) is an energy integral with the total energy being zero. So the number of constants is reduced to three. Also a constant in ω is irrelevant since the equations of motion do not involve ω (this reflects the fact that such a constant can be absorbed in the rescaling of coordinates). Thus there really are only two constants (say $\phi(0)$ and $\omega'(0)$) that can be then chosen to satisfy the matching conditions. As explained in [21] when the first order equations in terms of W are being solved one would replace $\omega'(0)$ by the integration constant coming from integrating (18). Thus with one brane a flat solution can be obtained without any fine tuning. Such a one brane solution we believe is unlikely to arise say from string theory since the brane typically carries some charge which would mean that the fifth dimension would have to be non-compact. This may however be a way of getting the scenario of the second paper of [19], but with the exponential potential for ϕ that naturally arises in string theory, one gets a logarithmic behaviour for the warp factor ω [23][5],[6][7] rather than the linear behaviour required in [19]. Later on we will come back to the scenario of [19] in a situation where the modulus field has been integrated out from the low energy theory.

When there are two branes there is another pair of matching conditions to satisfy, but also there is another parameter namely the value $u = R$ at which the new brane is situated (so in the IIB example this would be size of the orbifolded fifth dimension). Then we have an additional pair of conditions,

$$\begin{aligned} 3\omega'|_{u=R-} &= +\frac{1}{2}\kappa^2 T_0(\phi)|_{u=R-} \\ Z(\phi)\phi'|_{u=R-} &= -\frac{1}{2}\kappa^2 \frac{dT_0}{d\phi}(|_{u=R-} \end{aligned} \tag{22}$$

From the energy constraint (the first equation of (15) we also have

$$Z^{-1}\left(\frac{\kappa^2}{2}\frac{dT_0}{d\phi}\right)^2|_{u=0+} - \frac{4}{3}\left(\frac{\kappa^2}{2}T_0\right)^2|_{u=0+} = V|_{u=0+} \tag{23}$$

[7]In the last two references the singularity in such a metric is interpreted as a point where the space is to be cut off. However it is not entirely clear to us how this can arise from a microphysical theory such as string theory.

There is of course a similar equation at the point $u = R$ but this is not independent. Once we have a solution to the equations of motion and the matching conditions this will be automatically satisfied. In general the last equation will have a discrete set of solutions for $\phi(0)$.

Thus there is one extra condition and to satisfy that requires a fine tuning, either of the brane tension or of the potential [21]. However here too fine tuning can be avoided if we make at least one coupling constant in the potential dynamical, i.e. an integration constant. This is easily done if the bulk potential comes (at least partly) from the five dimensional analog of the F^2 term in (1) or (9). In this case, after solving the F equation of motion as in the discussion in section I and substituting to get an effective action without F, one gets a potential for ϕ which depends on the integration constant μ as in the discussion after (1). In the string theory context however (as we shall discuss in more detail later on) the brane is a source for the five form field and so μ is quantized. However in that case as we shall see later, if one considers (squashed) sphere compactifications one gets an additional adjustable parameter which is an integration constant of the ten dimensional theory. This is of course fine tuning from the from the point of view of the five dimensional theory, but not from the fundamental ten dimensional point of view.

There are several different cases one may consider.

a) Supersymmetry is unbroken both in bulk and on brane(s).
b) Supersymmetry is preserved in bulk and broken on the brane(s).
c) Supersymmetry broken in both bulk and brane(s).
d) Dilaton (and all other moduli) are fixed at the string scale

Let us discuss in turn the above cases.

a) Let us for example take a case which can come from type IIB orientifold constructions compactified to five dimensions (say on a torus or an orbifold). The low energy effective action contains a term $\int U(\phi)F_5\wedge^* F_5$, where ϕ is the five dimensional dilaton. If one solves the equation of motion for the corresponding gauge field as in (10) then one effectively gets a potential $V(\phi) = \mu^2 U^{-1}(\phi)$. Thus in the type IIB case, $V = \mu^2 exp(\frac{5}{3}\phi)$.In this case $T_i = \tau_i e^{\frac{5}{6}\phi}$ where τ_i is the (constant) brane charge. If we substitute this in (23) we see that in fact the $\phi(0)$ dependence drops out and the equation is satisfied if $\mu^2 = \frac{13}{9}(\frac{\kappa^2}{2})^2\tau^2$. It should be noted that in this case even with one brane one needs the non-zero solution to the F equation (i.e. (10). This is to be expected since as one crosses the brane the F field must change by the number of branes times the charge on a brane and in the supersymmetric case this charge is related to the tension.[8] In the two brane case there is no determination of the distance between the branes as is to be expected.

b) This case is more interesting. Now supersymmetry is broken on the brane and so the tension need not be as in a). In this case one would expect (23) to determine $\phi(0)$ and the matching conditions will determine the other two integration constants. In the first order formalism one of the integration constants will be the value of (say) $W(\phi(0))$.

[8] This might be thought of as being similar to that discussed in [24] and [25]. In fact in the latter paper it is pointed out that this case does not exist in source free gauged supergravity. However our case is somewhat different in that we have explicit sources. So it is not completely clear to us that the arguments of [25] apply.

If we work in the second order formalism after fixing $\phi(0)$ as above the two constants to be determined by the the two matching conditions (21) are $\phi'(0)$ and $\omega'(0)$). Thus one would indeed obtain (by choice of integration constants) a flat brane in 4D without fine tuning. When there is a second brane however, as we discussed earlier, there is one extra parameter (the distance R), but two more matching conditions to satisfy, and so we need to have a dynamical bulk cosmological constant.

c) In this case the bulk potential will also get renormalized but as far as the existence of flat brane solutions without fine tuning goes, there are no qualitatively new features compared to b).

d) This case we believe is quite interesting since it seems very likely that the moduli are fixed at (or close to) the string scale.[9] This as we mentioned earlier would correspond to the original Randall-Sundrum scenario [19]. This is possible in a situation in which stringy non-perturbative effects give a potential to all the moduli which should therefore be integrated out from the low energy effective action. In the absence of a string field theory, it is difficult to make precise statements . It is hard to see how the integration constant we want would arise if all moduli are fixed at the string scale since all we know how to deal with are low energy actions. Nevertheless perhaps one can make some educated guesses about this case too.

A possible string theoretic construction for the scenarios in cases b) and c) and possibly d) with one modulus field (the breathing mode) remaining in the action, may run as follows. Consider type IIB compactified to five dimensions on S^5 as in [27] (section 2.4). We assume that the ten dimensional dilaton φ is fixed by string scale dynamics, but we keep the breathing mode that arises in the compactification, in the action. The relevant part of the low energy effective action is

$$S = \int d^5x\sqrt{-G}(R - (\nabla\phi)^2) - 8m^2 e^{8\alpha\phi} + e^{\frac{16\alpha}{5}\phi}R_5) \tag{24}$$

In getting the above from the 10 D action (or equations of motion) of IIB supergravity, the ansatz

$$ds_{10}^2 = e^{2\alpha\phi}ds_5^2 + e^{2\beta\phi}ds^2(S^5)$$

with $\alpha = \frac{1}{4}\sqrt{\frac{5}{3}}$, $\beta = -\frac{3}{5}\alpha$ has been made. R_5 is the Ricci scalar of the compact 5-manifold (in this case S^5) and is an unconstrained positive constant. The ansatz for the self-dual 5-form, is

$$F_5 = 4me^{8\alpha\phi}\epsilon_{(5)} + 4m\epsilon_{(5)}(S^5) \tag{25}$$

where m is an integration constant.

At this point let us show out how the quantization of m mentioned earlier comes about if this scenario emerges from string theory. In this case the sources of the F_5 are D_3 branes (and/or orientifolds). The consistency of the the coupling of these objects leads to the standard Dirac quantization rule,

$$\tau_3 \int_{S_5} F_5 = 2\pi n$$

[9] For a discussion of this with references to earlier work see [26].

where n is an integer. Using the ansatz (25) for F_5 then gives $\tau_3 4m\pi^3 r^5 = 2\pi n$ where r is the radius of S_5. This then gives us (after using the D3-brane tension formula $\tau_3 = 2\pi M_s^4$)

$$m = \frac{n}{4\pi^3 M_s^4 r^5} = \frac{nM_s}{4\pi^3 \hat{r}^5} \tag{26}$$

where we have introduced the dimensionless parameter $\hat{r} \equiv rM_s$. Also the Ricci scalar of S_5 may then be written as $R_5 = \frac{20M_s^2}{\hat{r}^2}$. Then the potential in (24) may be rewritten

$$\begin{aligned} V &= M_s^2 \left[\frac{n^2}{2\pi^6 \hat{r}^{10}} e^{8\alpha\phi} - \frac{20}{\hat{r}^2} e^{\frac{16}{5}\alpha\phi} \right] \\ &= M_s^2 \hat{r}^{\frac{10}{3}} \left[\frac{n^2}{2\pi^6} e^{8\alpha\tilde{\phi}} - 20 e^{\frac{16}{5}\alpha\tilde{\phi}} \right] \end{aligned} \tag{27}$$

where in the last line of the above equation we put $\alpha\phi = \alpha\tilde{\phi} + \frac{5}{3}\ln\hat{r}$.

The minimum of this potential is given by $e^{\frac{24}{5}\alpha\tilde{\phi}_0} = \frac{16\pi^6}{n^2}$. As discussed in [27] the 5D action allows a AdS_5 solution with the cosmological constant being an integration constant, with the breathing mode being fixed at $(\tilde{\phi}_0)$. The potential at this point gives a cosmological constant $\Lambda = V(\tilde{\phi}_0) = -(2\pi)^4 \frac{16}{n^2} M_s^2 \hat{r}^{\frac{10}{3}}$. It should be noted that this five dimensional cosmological constant is dependent on the compactification parameter $\hat{r}$ and can be adjusted even though there are no adjustable constants in the 10 dimensional action.

Let us now compactify one of the spatial dimensions on S^1/Z_2 as before. In the original string theory scenario we would have $D3$ branes and orientifolds distributed over the five sphere but we will just consider the effective theory in five dimensions with two branes sitting at the ends of the compact fifth dimension giving us the scenario we had earlier.

The main focus of our discussion is going to be on how to get a two -brane scenario without fine tuning of parameters in the 10 dimensional Lagrangian. This will require that we keep the breathing mode $\tilde{\phi}$ as a dynamical field that is not sitting at the minimum of the potential. However before we do that let us see whether we can get a Randall-Sundrum [19] type scenario with the scalar breathing mode integrated out. For this we need to assume that some string non-perturbative effects fix the breathing mode at some high scale so that in the effective low energy five dimensional theory it has been integrated out just like the ten dimensional dilaton. Then we would have a bulk cosmological constant which will get contributions from the string scale effects as well as from V. The latter will of course not necessarily be fixed at $V(\tilde{\phi}_0)$ since it will be primarily determined by string effects however the important point is that such a contribution will be $\hat{r}$ dependent and hence will be adjustable. Making the mild assumption that this effective cosmological constant is negative we put $V = -\frac{\mu^2}{4}(\hat{r})$) in the first equation of (15). This then becomes $-12\omega'^2 = -\frac{\mu^2}{4}$ giving $\omega' = \pm\frac{\mu}{4\sqrt{3}}$, so that $\omega = -\frac{\mu}{4\sqrt{3}}|u|$. In the last equation we have used the Z_2 symmetry so as to obtain a warp factor that decays exponentially from the origin[19] on both sides. Using the matching condition (21) then

gives $\kappa^2 T_0 = \frac{3\mu}{2}$. The point is that this condition can be satisfied without fine tuning of the tension since the bulk potential is a function of the adjustable compactification constant $\hat{r}$. If the size of our compact direction is taken to infinity then we have the RS2 scenario. On the other hand if this dimension is finite then we need a second brane at $u = R$ necessarily its tension is negative $T_1 = -\frac{3\mu}{2}$. This of course is then a fine tuning.

There are several points that should be noted in this calculation.

- In the absence of the modulus field there is no flat one brane solution without fine tuning (as in [19]) or having a dynamical cosmological constant as in the above discussion. Indeed in the latter case there is (perhaps) a theory of confined gravity as in the second paper of [19] but obtained now without fine tuning of the fundamental (ten-dimensional) Lagrangian.

- In the RS1 scenario the distance R is now a free parameter (adjusted to a value that "explains" the gauge hierarchy in [19]) and is not fixed by the dyanmics. Indeed the scalar field was introduced in [20] in order to stabilize the value of R. However this requires a tuning of a parameter in the potential in order to obtain the "right" value. So unless this value of the parameter in the potential has a natural explanation there is no particular advantage to this.

- In the two brane case the so-called visible brane (on which the standard model is supposed to live) has negative tension. Also since the dynamical bulk cosmological constant tracks the brane tension at the origin as it changes with RG scale the only way (without fine tuning) for a two brane solution to be viable is for the RG flow of the visible brane to be the same in magnitude though opposite in sign as on the other brane. It is not clear to us how to achieve this in a natural way. Thus this still requires fine tuning.

Let us now get back to the main point of this discussion. To discuss a two brane case (in cases b),d) above) with at least one modulus field dynamical we should look at a IIB orientifold. Now the fifth dimension is an interval S_1/Z_2 with 16 orientifold fixed planes at the fixed points $u = 0$ and $u = R$. One also needs to introduce D-branes in order to cancel tadpoles[10]. In the presence of D-branes and orientifold planes that are charged under the F_5 field we have (as in [28]) a discontinuity in m by an amount equal to the brane charge/tension at the position of the brane. Imposing also the Z_2 symmetry the constant m in (24) would be fixed in terms of the brane charge as in [28]. This is of course just another way of verifying the quantization discussed earlier. When supersymmetry is broken however the brane tension would get renormalized so that the supersymmetric relation between tension and charge will be lost. Nevertheless the constant $\hat{r}$ can adjust itself now to track the brane tension. In addition (assuming it is not fixed at the string scale by stringy effects as in the previous example) we have a modulus field ϕ as in the discussion at the begining of this section, to supply an addtional integration constant so that one may have solutions with two flat branes.

[10] Indeed such a model is T-dual to the type IA theory discussed in [28].

Let us give some more details of this scenario. The effective five dimensional theory in the presence of branes is given by (after putting in also the value of the five D Newton constant)

$$\begin{aligned} S &= 2\pi^4 M_s^3 \hat{r}^5 \int d^5x\sqrt{g_5}\left(R - \frac{1}{2}(\partial\tilde{\phi})^2 + M_s^2 \hat{r}^{\frac{10}{3}} U(\tilde{\phi})\right) \\ &+ \frac{2\pi M_s^4}{\pi^3}\left\{\int_{u=0} d^4x\sqrt{g_4}T_0(\tilde{\phi})\int_{u=R} d^4x\sqrt{g_4}T_1(\tilde{\phi})\right\}. \end{aligned} \tag{28}$$

In the above we have written the potential $V = M_s^2 R^{\frac{10}{3}} U(\tilde{\phi})$ where U is independent of $\hat{r}$ (see (27)). Let us first look at what might happen in the supersymmetric case. Here in analogy with the case discussed by Lucas et al [29] for the Supersymmetry preserving compactification of M theory on $\frac{S^1}{Z_2} \times CY_3$ we expect the BPS equations

$$T_0(\tilde{\phi}) = -T_1(\tilde{\phi}) = \pi^3 \hat{r}^{5/3} W(\tilde{\phi})$$

where

$$U(\tilde{\phi}) = \left(\frac{\partial W}{\partial\tilde{\phi}}\right)^2 - \frac{2}{3}W^2$$

. (Note that apart from normalization this is the same W as before). In fact these BPS conditions have been justified for this system very recently in [**?**],[**?**]. In this case it is easily seen that the parameter $\hat{r}$ drops out of the matching conditions which however can be satisfied for arbitrary inter-brane distance R and integration constants. This is exactly like what happens in the M theory case investigated by Lukas et al. [**?**].

In the broken supersymmetric case however the situation is quite different. For instance we may imagine now is that the compactification is on a squashed sphere so that the supersymmetry is $N = 1$ which is then broken by for instance by gaugino condensation effects on one or other brane. Now the tension will be renormalized from its BPS value so that one would expect

$$T_i(\tilde{\phi}) \to \pm\pi^3 \hat{r}^{\frac{5}{3}} W(\tilde{\phi}) + \epsilon\psi_i(\hat{r}, \tilde{\phi})$$

Here the function ψ is dependent on the details of the low energy field theories on the branes. Thus now $\hat{r}$ will not drop out of the matching conditions and indeed we need to adjust the two integration constants $\tilde{\phi}(0), A'(0)$ the inter-brane distance R as well as the compactification parameter $\hat{r}$ in order to get a flat four-dimensional brane world.

A detailed discussion of such models will be given in a forthcoming paper [32].

4 Conclusions

Let us first discuss the results of [5],[6]. From our discussion it should be clear that the reason that flat (one) brane solutions are obtained (without fine tuning) in these works is that integration constants have been chosen to ensure the existence of such solutions.

Of course since these authors do not discuss two brane solutions they do not need the F_5 field or the Ricci non-flat compactification that we have introduced. However as we argued (and is indeed implied by the work of de Wolf et al [21]) one flat brane solution is obtained in the presence of a dynamical scalar field by choosing the integration constant $\phi(0)$ appropriately. It does not depend on the particular form of the brane tension $T(\phi)$ as seems to be implied in [5]. Indeed this is just as well since the form of this function can change under renormalization effects on the brane. The fact that *only* a flat brane is allowed for a particular form of this function (see equation (14) of [5] and section (3.2) of [33]) therefore is not a RG invariant statement. Quantum effects of the standard model in a background metric yields both a cosmological constant as well as curvature terms (as in our (7)). The latter will necessarily modify these arguments.

The main conclusion of the present work is that one can indeed obtain flat branes (and in particular zero cosmological constant in the brane containing the standard model) without fine tuning, but it involves a choice of integration constants/compactification parmeters. In this respect these theories have the same problem that bedevilled those of references [8],[11] [10],[12]. It is useful to review this issue briefly. The point is to show that the particular integration constant(s) that leads to a zero cosmological constant gets chosen because it is the most probable one. To show this Hawking used a Euclidean quantum gravity argument according to which (see also section VIII of Weinberg's classic review [14]) the probability for the occurence of a value μ for the integration constant was given by $P(\mu) \propto \exp(-\Gamma_E[\psi_c])$ where Γ_E is the Euclidean quantum effective action (essentially our equation (9) Wick rotated to a Euclidean metric) and the ψ_c are the values of all the fields evaluated at an extremum of Γ. The Euclidean (effective) action for a D dimensional theory after setting all other fields but the metric to their quantum ground state values as above would take the form, (setting the D dimensional Planck mass equal to one) $\Gamma_E = -\int \sqrt{G}(R - \Lambda)$. From the Einstein equation we have $R = \frac{D\Lambda}{(D-2)}$. Substituting this into the Euclidean action gives $S_E = -\frac{2V_D}{D-2}\Lambda$ where V_D is the volume of Euclidean D space. If Λ is positive then the space is S_D and its volume is $V_D = \frac{a^2}{\Lambda^2}$ so that the action becomes $S_E = -\frac{2V}{(D-2)\Lambda}$. Thus the probability distribution becomes

$$P(\mu) \propto e^{-\Gamma_E[\psi_c]} = e^{+\frac{2V}{(D-2)\Lambda}}. \tag{29}$$

This would imply that the probability was peaked at $\Lambda \to 0+$. [11]

In our case it is not clear whether an analog of Hawking's argument would work[12]. However one would think that one should apply the argument to the five (or ten?) dimensional theory since that is the action one is starting from. But the integration

[11] It was pointed out by Duff [34] that if one substitutes the solution for F into Einstein's equation and then infers the effective action from which it comes, then one finds in fact that the (Euclidean) action is positive near $\Lambda = 0$ so that this value is actually disfavoured! Nevertheless it was claimed in [35] that the correct action has a boundary term that does not affect the equation of motion and its inclusion will reestablish Hawking's result. I wish to thank R. Bousso for pointing out this reference to me. On the other hand as pointed out by M. Duff (private communication) Hawking's Euclidean space no-boundary action should not of course have a boundary contribution! The situation therefore remains murky.

[12] Hawking's argument may work also in the case of unimodular gravity [36].

constants must get chosen so that it is the four dimensional theory that has to have zero cosmological constant. At this point it is not clear to us whether a version of this argument can be used to justify the choice of integration constants.

Note added: While this paper was being prepared for submission, a paper which (inter alia) makes comments related to ours about the one brane case of [5],[6], appeared as an e-print [37].

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